

# Margin Measurements in Optical Amplifier Systems

Neal S. Bergano, F. W. Kerfoot, and C. R. Davidson

**Abstract**—The margin, or the difference between the received signal-to-noise ratio (SNR) and the SNR required to maintain a given bit error ratio (BER), is important to the design and operation of optical amplifier transmission systems. A new technique is described for estimating the SNR at the receiver's decision circuit when the BER is too low to be measured in a reasonable time. The SNR is determined from the behavior of the BER as a function of the decision threshold setting in the region where the BER is measurable. We obtain good agreement between the BER predicted using the measured SNR value and the actual measured BER.

## I. INTRODUCTION

THE bit error ratio (BER) in an optical amplifier transmission system is set by the electrical signal-to-noise ratio (SNR) of the data signal at the decision circuit. Therefore, SNR is a natural figure of merit for transmission performance and system health. The margin in an optical amplifier system is the decibel difference between the received SNR and the SNR required to maintain the system error rate specification. The received SNR, and therefore the margin, is set by optical noise and waveform degradations accumulating over the *entire* length of the system. In this letter, we propose a method of measuring margin in an optical amplifier system, by estimating the SNR at the receiver's decision circuit, when the BER is too small to be directly measured. We observe good correlation between measured and calculated BER for a 5 Gb/s NRZ signal through a 4500 km optical amplifier system.

## II. SNR IN OPTICAL AMPLIFIER SYSTEMS

In an optical amplifier system, the SNR at the decision circuit is degraded by optical noise, fiber chromatic dispersion, polarization mode dispersion, and fiber nonlinearities. While the exact probability density function for optical noise is not exactly Gaussian [1], a Gaussian approximation can lead to close BER estimates [2]. The  $Q$  factor [3] is the signal-to-noise ratio at the decision circuit in voltage or current units, and is typically expressed by

$$Q = \frac{|\mu_1 - \mu_0|}{\sigma_1 + \sigma_0} \quad (1)$$

where  $\mu_{1,0}$  is the mean value of the marks/spaces rail,

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The authors are with AT & T Bell Laboratories, Holmdel, NJ 07733.  
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and  $\sigma_{1,0}$  is the standard deviation. A voltage histogram down the center of the eye can be measured with a digital sampling oscilloscope to estimate  $Q$ . This technique fails, however, to give good correlation between the measurement of  $Q$  and the BER, since the variation seen around each rail represents a mix of pattern effects, such as intersymbol interference (ISI) and noise. Such effects in turn artificially broaden the estimates of  $\sigma_{1,0}$ , thus giving erroneous results. In addition, this method operates on a limited set of bits (i.e., the data arrives at 5 Gb/s, while the oscilloscope's analog-to-digital converter samples at 10–100 kHz).

Alternatively, the voltage histograms can be made at a specific point in the pattern as opposed to the data eye. This eliminates the pattern effects from the measurements of  $\sigma_{1,0}$ , but yields a potentially inaccurate measure of  $\mu_{1,0}$ . Also, this approach has the drawback of recording even fewer bits than the measurement in the eye, and it is not practical in a real transmission system, where the data bits are random. Our new technique avoids these problems by using the decision circuit to probe the rails of the eye, thus using every bit in the data stream. Moreover, it includes the ISI present in the regenerator's linear channel as well as that generated in the system from dispersion and fiber nonlinearity.

## III. MEASUREMENT TECHNIQUE

The  $Q$  factor is measured by recording the BER versus decision level down the center of the eye (i.e., a fixed timing phase). The equivalent mean and sigma of the marks and spaces are determined by fitting this data to a Gaussian characteristic, given by [4]

$$\text{BER}(D) = \frac{1}{2} \left\{ \text{erfc} \left( \frac{|\mu_1 - D|}{\sigma_1} \right) + \text{erfc} \left( \frac{|\mu_0 - D|}{\sigma_0} \right) \right\} \quad (2)$$

where  $\mu_{1,0}$  and  $\sigma_{1,0}$  are the mean and standard deviation of the mark and space data rails,  $D$ , is the decision level, and  $\text{erfc}(x)$  is a form of the complementary error function given by:

$$\text{erfc}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\beta^2/2} d\beta \approx \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} \quad (3)$$

where the approximation is nearly exact for  $x > 3$ . Here the  $\mu_{1,0}$  and  $\sigma_{1,0}$  are not the physical values in the eye,

rather they are equivalent values used to fit the data for the purpose of estimating  $Q$ .

The  $Q$  factor is calculated as follows: using the  $\mu$  and  $\sigma$  of each rail in a fashion similar to (1), the data are divided into two sets that have the measured BER dominated by the marks rail and spaces rail. The raw data are separated at the point of minimum error rate for measurable BER's, or at any value of  $D$  that yields error-free performance, for cases where the SNR is high. Each data set is fitted to an ideal curve, assuming Gaussian noise statistics, to obtain an equivalent mean and sigma for the positive and negative rail. Equation (2) naturally separates into errors dominated by mark errors and space errors. Once separated, the BER is a simple expression given by a single  $\frac{1}{2} \operatorname{erfc}(\cdot)$  function. Each set of BER data is passed through an inverse error function, and then a linear regression is performed with the decision levels  $D_i$ . The equivalent  $\sigma_{1,0}$  and  $\mu_{1,0}$ , are given by the slope and intercept of the linear regressions. For ease of computation, the inverse  $\frac{1}{2} \operatorname{erfc}(\cdot)$  function is performed by first taking the logarithm of the BER.  $\log(\frac{1}{2} \operatorname{erfc}(\cdot))$  is a smooth one-one function that can be inverted by many numerical methods, or more simply by using a polynomial fit:

$$\left\{ \log \left( \frac{1}{2} \operatorname{erfc}(\cdot) \right) \right\}^{-1}(x) \approx 1.192 - 0.6681x - 0.0162x^2 \quad (4)$$

where  $x$  is  $\log(\operatorname{BER})$ , and (4) is accurate to  $\pm 0.2\%$  over the range of BER's from  $10^{-5}$  to  $10^{-10}$ . The optimum decision level  $D_{\text{opt}}$  is determined from  $\mu_{1,0}$  and  $\sigma_{1,0}$  as the cross point for the two Gaussian probability density functions.<sup>1</sup> The calculated BER is given by (2) evaluated at  $D_{\text{opt}}$ , and to a good approximation is given by  $\operatorname{erfc}(Q)$ .

#### IV. RESULTS

The BER was measured at 5 Gb/s using an NRZ transmitter regenerator pair similar to that used in previous experiments [5]. The transmitter consists of a CW DFB laser source and a Mach-Zehnder intensity modulator, with a rise time of 50 ps. The regenerator uses an optical amplifier automatic gain control amplifier (AGC) front-end, a 2 nm bandpass filter, a p-i-n diode O/E converter, phase-locked loop timing recovery, and a decision circuit with a variable decision level. The  $10^{-9}$  sensitivity of the transmitter/regenerator pair is  $-36.4$  dBm at 5 Gb/s using a  $2^{15} - 1$  word. Measurements were made in the back-to-back configuration and through a 4500 km amplifier chain [5].

Fig. 1 shows typical measured data for the logarithm of the BER versus the decision threshold in the decision circuit, and the solid line shows the best fit of (2). In this measurement, the optical power into the receiver was about 3.5 dB over the  $10^{-9}$  sensitivity point. The BER was measured over one second intervals and was considered valid if at least five errors were recorded; thus, the minimum measured BER was  $10^{-9}$ . The maximum BER mea-

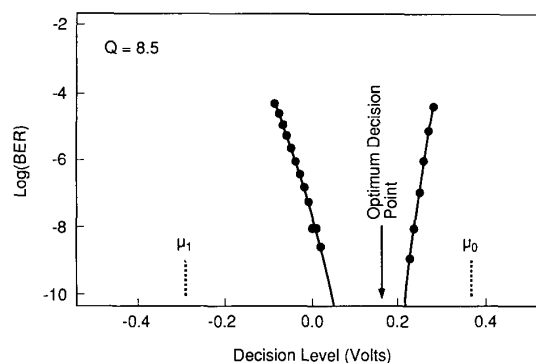


Fig. 1. Bit error ratio versus decision threshold for back-to-back operation.

sure was about  $10^{-5}$ . The  $Q$  factor was determined to be 8.5 in linear ratio. The down arrow shows the predicted optimum decision threshold, and the two vertical segments show the equivalent  $\mu_i$  for the marks and spaces rail.

The  $Q$  factor and BER were measured for different optical powers into the receiver. Changing the optical power into the receiver in turn changed the SNR at the input to the optical-to-electrical converter (O/E), since the receiver had an optical amplifier AGC front-end that maintained constant total power into the O/E. The circles in Fig. 2 show the measured BER versus  $Q$  factor for the back-to-back operation, and the broken line shows the ideal characteristic. Since the  $Q$  factor is estimated from the BER as measured in the range of  $10^{-5}$  to  $10^{-9}$ , the data necessarily matches for a BER greater than  $10^{-9}$ . The interesting points are those with a BER less than  $10^{-9}$ , where the data show a good match to the prediction down to BER's as low as  $10^{-13}$ . The good match between measurements and calculations with the 4500 km amplifier chain gives us confidence that the technique works in the presence of realistic noise and waveform distortion. The BER's were measured at the optimum decision threshold predicted from the fitting algorithm. The good fit to the data in Fig. 2 also shows that the Gaussian approximation predicts the proper decision threshold for measurable BER's.

This measurement technique allows us to make accurate predictions of the margin available in an optical amplifier system. For example, Fig. 3 shows the measured  $Q$  factor versus the received optical power (ROP) into the receiver and includes the data shown in Fig. 2. Similar to electrical SNR, the  $Q$  factor in the figure is expressed in decibels as  $Q(\text{dB}) = 20 \log(Q)$ . For an ROP greater than 2 dB over the sensitivity point, it is impractical to measure the BER; however,  $Q$  is easily measured and shows continued increases with an ROP for the back-to-back case, until the maximum input power of  $-6.7$  dBm is reached. The margin or the difference in  $Q$  value for  $10^{-9}$  BER operation and the maximum input power was 18.5 dB. If more transmitter power were available, the measured  $Q$

<sup>1</sup> Assuming equal probability for a mark or space.

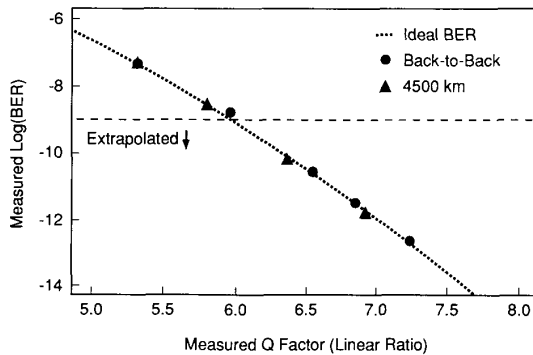


Fig. 2. Measured bit error ratio versus  $Q$  factor for back-to-back, and 4500 km operation.

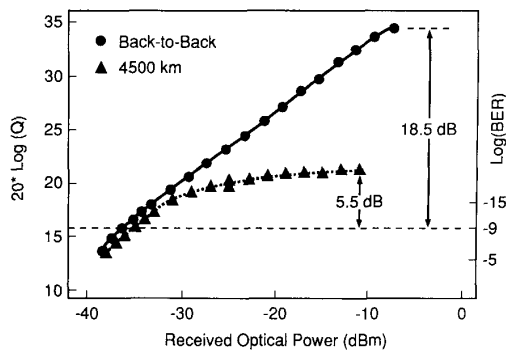


Fig. 3. Measured  $Q$  factor versus received optical power, back-to-back and 4500 km.

value would eventually reach a constant value corresponding to the SNR of the transmitted signal. With the 4500 km amplifier chain, the  $Q$  factor reached a steady state value of 21.2 dB, for a margin of 5.5 dB.

#### V. CONCLUSIONS

We have described a technique for measuring the signal-to-noise ratio at the decision circuit (or  $Q$  factor) of an optical amplifier transmission system. The measured system bit error ratio is accurately predicted from the SNR measurement. The technique can be used to adjust the decision point of the regenerator in the terminal adaptively, since the measurement predicts the optimum threshold, and the results can be extended to select the optimum phase. This measurement should apply equally well to loop experiments using NRZ or soliton transmission.

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